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13. ABSTRACT (Maximum 200 words)

This final report briefly describes research results on a theory of gain scheduling for flight control applications that were obtained by the Principal Investigator and his students over the three-year period of support. Results reported include development of a basic theory of gain scheduling in nonlinear systems, solution of output regulation problems based on an exogenous system assumption for the exogenous signals (including disturbance and scheduling signals), initial development of methods to alleviate performance degradation in the case of rapidly-varying scheduling signals, and an exploratory application of these results to an autopilot design example. Publications describing the results in detail are listed.

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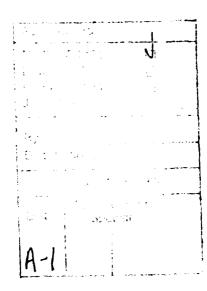# ANALYTICAL FOUNDATIONS OF GAIN SCHEDULING

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Final Technical Report
under
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Bolling Air Force Base, Washington, DC 20332

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1. Research Objectives

The overall goal of this research program was to develop the basic concepts and analytical foundations for gain scheduling methods that are widely used in the design of flight control systems. Major topics include an investigation of the formulation of gain scheduling design methods within the framework of modern nonlinear control theory, characterization of the constraints that must be satisfied by the scheduled control law at intermediate operating conditions, development of scheduling methods that incorporate these constraints, and the application of new stability results to the analysis of gain-scheduled systems.

2. Research Status

Research accomplishments under this grant divide into four main categories: (1) creation of a basic theoretical framework within which gain scheduling issues for nonlinear control design can be addressed; (2) solution of nonlinear control problems involving exogenous signals (such as scheduling signals) under the assumption that the exogenous signals are generated by a known exogenous system; (3) development of approaches to alleviate performance degradation when the scheduling signal is not slowly-varying; and (4) an exploratory application of our developing theory to a proto-type missile autopilot design problem. These accomplishments are briefly discussed below in conjunction with the corresponding publications under the grant, as listed in Section 3.

In publication [1], "Analytical Framework for Gain Scheduling," a basic theoretical perspective is proposed for the gain scheduling problem as it is currently viewed in practice. This involves the design of fixed linear controllers at fixed operating conditions, and then the process of devising a variable-gain controller that reduces to the respective fixed controller when the plant is operated near the respective fixed condition. When certain prescriptions are followed, it is shown that basic performance and overall stability can be maintained in the face of slowly-varying scheduling variables. However it is clear, both from our analysis and from practice, that a number of problems arise if the scheduling variable varies too rapidly, that is, if the operating condition changes too rapidly. Even if stability is preserved, the instantaneous error in a tracking or regulation context is driven by the time derivative of the scheduling signal, so that performance degrades.

We have investigated general versions of the nonlinear servomechanism problem, also called the output regulation problem, which are related to gain scheduling problems. The key assumption is that the exogenous signals, including scheduling signals treated essentially as measured disturbance signals, are generated by known exogenous systems, and the results are based on certain invariant manifolds, which we call zero-error manifolds. Publications [3] and [4] contain the detailed results. In [3], "Stabilization on Zero-Error Manifolds and the Nonlinear Servomechanism Problem," we begin with the assumed existence of the zero error manifold and provide a self-contained and relatively elementary proof of stability for a solution of the problem based on an extended-linearization control design. One additional feature of our analysis is that certain types of unbounded exogenous signals (including ramps) are permitted. This work is further developed in [4], "An Approximation Approach for the Nonlinear Servomechanism Problem," which provides an effective way to address the calculation of 'approximate zero-error manifolds,' and compute control laws that approximately solve the problem. Of note here is that we prove that higher-order approximations yield better solutions in terms of the order of the error.

Related research accomplishments are reported in [2] and [5]. In [2] an approach to overcoming stability obstructions in the design of stable noninteracting control systems is described. Again the formulation of the problem is similar to our formulation of the gain scheduling problem, and the results of this work may become important in the design of gain scheduled control to avoid cross-channel coupling in autopilots. In [5] we describe a high gain approach to tracking that arose as an offshoot to our work on gain scheduling. In a manner not unlike the gain scheduling problem, the instantaneous error in, for example, tracking a reference input, depends on the rate of change of the input. We show that in the linear case this can be formulated as a trade-off between instantaneous error and and the input time derivative, and high-gain feedback can be used to achieve desired performance. However, because of difficulties with high-gain feedback in the nonlinear context, we have not pursued this topic further.

Recent efforts concentrated on gain scheduling in the presence of fast scheduling variables. We have been seeking approaches to the design of gain scheduled control laws that will be less prone to performance deterioration when the scheduling variable is not 'slowly varying.' By using the derivative of the scheduling variable in the control law − a feasible practice in many situations − we have been able to prove that output regulation can be achieved as long as the second derivative of the scheduling variable is small ('slowly accelerating'). [8] The methodology is based on robust control theory, and the main disadvantages of the approach are in the restrictive hypotheses, and the complexity of the control law.

Because of these disadvantages we have taken another approach to the fast-scheduling-variable problem, though again making use of the derivative of the scheduling variable in the control law. Preliminary results on academic examples are promising, though we have not succeeded in obtaining a mathematical analysis that demonstrates the properties of the control law to a satisfactory extent. That is, the control design seems better than we have been able to prove so far. We plan to continue the analysis work, and also explore performance of the technique on more realistic examples. (See the discussion of the autopilot design problem below.)

We have also reformulated our approach to gain scheduling in the context of scheduling high-order linear dynamic controllers of the type that result from modern robust linear design methods (H_∞ and mu-synthesis). This gives somewhat different compatibility conditions for rational scheduling than were discussed in the context of state feedback in [1]. This work is particularly important in view of the increasing application of the modern linear methods in autopilot design, and also interfaces with the design project discussed below. As we gain experience it is increasingly clear that the most effective design method for the typical autopilot design problem is to use robust linear control methods to meet performance specifications at various operating conditions while maintaining robustness to 'fast, small' nonlinearities. Then use a rational gain scheduling technique to handle 'slow, large' nonlinear effects while preserving performance. This combination makes use of the great strength of modern linear design methods in addressing complex performance requirements, and avoids the limited capabilities of current nonlinear approaches in this regard. (Namely the existing nonlinear theory is limited to very specific types of control objectives − mainly stabilization, output regulation, and decoupling objectives. The complex, conflicting performance requirements in a typical autopilot design go well beyond these objectives.)

We have completed a feasibility study for these results on a pitch axis autopilot design problem formulated by colleagues at the Johns Hopkins Applied Physics Laboratory. The

objective is to track commanded normal acceleration over an operating envelope that includes significant variation in Mach number and angle of attack. This variation results in significant change in aerodynamic coefficients. The performance requirements are realistic, and sufficiently stringent (including robustness criteria) that linear designs have been unsuitable. The paper [7] reports our results on gain scheduling robust linear designs at a small set of distinct operating conditions with respect to Mach number. Because our approach has been highly successful on this problem, interaction with the Applied Physics Laboratory on this topic has continued.

We have also been considering possibilities for using ideas from the exact linearization theory, including the theory of normal forms of nonlinear systems, in the context of gain scheduling. In [6] we have further developed the normal-form theory in a direction that will facilitate this effort. Again because of the complexity of the objectives in a typical autopilot problem, we do not expect immediate utility for this effort. However it was deemed important as a second-priority effort, in part to maintain contact with the large theoretical effort in exact linearization approaches.

Finally, I have included in the list of publications my new book [9] on linear system theory. While AFOSR did not directly support this effort, I thank AFOSR in the *Preface* for "support of research compatible with an attention to theoretical foundations."

3. Publications (Cumulative, from March 1990.)

1. W.J. Rugh, "Analytical Framework for Gain Scheduling," *IEEE Control Systems Magazine*, Vol. 11, No. 1, pp. 79 - 84, 1991. (Preliminary version appeared in the *Proceedings of the 1990 American Control Conference*, pp. 1688 - 1694.)

2. J. Huang and W.J. Rugh, "Approximate Noninteracting Control with Stability for Nonlinear Systems," *IEEE Transactions on Automatic Control*, Vol. 36, No. 3, pp. 295 - 304, 1991. (Preliminary version appeared in the *Proceedings of the 1990 American Control Conference*, pp. 651 - 656.)

3. J. Huang and W.J. Rugh, "Stabilization on Zero-Error Manifolds and the Nonlinear Servomechanism Problem," *IEEE Transactions on Automatic Control*, Vol. 37, No. 7, pp. 1009 - 1013, 1992. (Preliminary version appeared in the *Proceedings of the 1990 IEEE Conference on Decision and Control*, pp. 1262 - 1266.)

4. J. Huang and W.J. Rugh, "An Approximation Approach for the Nonlinear Servomechanism Problem," *IEEE Transactions on Automatic Control*, Vol. 37, No. 9, pp. 1395 - 1398, 1992.

5. D. Lawrence and W.J. Rugh, "Almost Perfect Tracking for Linear Systems," *Systems and Control Letters*, Vol. 17, pp. 281 - 289, 1991. (Correction, Vol. 18, No. 4, pp. 323-324, 1992.) (Preliminary version appeared in the *Proceedings of the 1991 American Control Conference*, pp. 626 - 629.)

6. D. Lawrence and W.J. Rugh, "Input-Output Pseudolinearization for Nonlinear Systems," submitted for journal publication. (Preliminary version appeared in the *Proceedings of the 1991 IEEE Conference on Decision and Control*, pp. 773 - 777.

7. R.N. Nichols, R.T. Reichert, W.J. Rugh, Gain Scheduling for H-infinity Controllers: A Flight Control Example," accepted for publication, *IEEE Transactions on Control Systems Technology, 1993.*

8. D. Guo and W.J. Rugh, "An Approach to Gain Scheduling on Fast Variables," *Proceedings of the 1992 IEEE Conference on Decision and Control*, pp. 759 763. (Journal version in preparation.)

9. W.J. Rugh, *Linear System Theory*, Prentice-Hall, Englewood Cliffs, NJ, 356 pages, 1993.

4. Personnel (Cumulative, from March 1990.)

Principal Investigator:

Wilson J. Rugh

Research Assistants (Graduate Students):

H. Jie: PhD completed, December 1990. Dissertation title: *On the Servomechanism and Noninteracting Control Problems for Nonlinear Systems.* (Currently on the staff of American GNC Corporation in California.)

D. Lawrence: PhD completed, June 1991. Dissertation title: *On the Analysis and Design of Nonlinear Control Systems Using Linearization Methods.* (Currently on the faculty of Ohio University.)

D. Guo: current PhD student.

N. Sureshbabu: current graduate student.

5. Interactions (Cumulative, from March 1990.)

A seminar on publication [3] was presented at the Systems Research Center, University of Maryland, College Park, MD, in April, 1990.

Publication [1] was presented as an invited paper at the 1990 American Control Conference, San Diego, CA, in May, 1990.

Publication [2] was presented at the 1990 American Control Conference, San Diego, CA, in May, 1990.

A seminar entitled "New Approaches in Control Theory and Application to Nonlinear Flight Control" was presented at AFOSR, Bolling AFB, in September, 1990.

Publication [3] was presented as an invited paper at the 1990 IEEE Conference on Decision and Control, Honolulu, HI, in December, 1990.

Publication [5] was presented at the 1991 American Control Conference, Boston, MA, in June, 1991.

A talk entitled "Gain Scheduling and High-Performance Autopilot Design" was presented at the Washington University AFOSR Workshop, St. Louis, MO, in August, 1991.

A talk entitled "Further Results on Gain Scheduling" was presented at the Workshop on Nonlinear Control of Articulated Flexible Structures, University of California, Santa Barbara, CA, in October, 1991.

Publication [6] was presented at the 1991 IEEE Conference on Decision and Control, Brighton, UK, in December, 1991.

Publication [7] was presented at the 1992 American Control Conference, Chicago IL, in June, 1992.

A seminar entitled "Gain Scheduled Controllers for Nonlinear Systems: What We Do When We Do What We Do" was presented at the Department of Electrical Engineering, University of Maryland, Baltimore County, in October, 1992.

Publication [8] was presented at the 1992 IEEE Conference on Decision and Control, Tuscon, AZ, in December, 1992.

Informal interaction with the flight control group at the Johns Hopkins Applied Physics Laboratory has continued, related to publication [7]. Plans are that the Laboratory will apply the gain scheduling approach we have developed to a major new autopilot design project.